2020 International Conference on Industrial Engineering, Applications and Manufacturing (ICIEAM)

Invariance of Modal Transformations of Electrical Values in Traveling Wave Fault Locator

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***Abstract*—The use of modal transformations in the traveling wave fault locator has difficulties associated with the need to calculate the conversion matrix from phase to modal values. This calculation requires considering the geometric features of the phases of the transmission line. Usually, the calculation of the conversion matrix simplifies and considers that the transmission line is symmetrical and fully transposed. It allows using well known modal transformations: Clarke, Karrenbauer and Wedepohl transformations which widely used in traveling wave fault locators. However, there is an unresolved issue: which one is optimal for traveling wave fault locator and gives the greatest accuracy for fault location.**

This article shows that Clarke transformation, Karrenbauer transformation, and Wedepohl transformation give identical wave characteristics and any of them can be used for fault location.

***Keywords*—modal transformation, Clarke transformation, Karrenbauer transformation, Wedepohl transformation**

I. Introduction

Phase values of currents and voltages contain two terms: aerial and ground (zero) modes [1]. Because these terms have different propagation speeds [2, 3], they arrive at different times at the point of installation of a traveling wave fault locator. This reduces the accuracy of determining traveling wave arrival time from phase values and, therefore, fault location.

To solve this problem wave propagation in transmission lines is considered on the basis of decompositions of phase values into independent air and ground modes [4]. Each mode is characterized by its own attenuation coefficient and wave propagation speed. Use of mode with less attenuation coefficient in traveling wave fault locator allows increasing accuracy of the time of arriving of traveling wave and fault location. To convert phase values to modal values Clarke transformation [5]–[8], Karrenbauer transformation [6], [9] and Wedepohl transformation [6], [10] is traditionally used in modern traveling wave fault locators. The question arises: which of them is optimal for traveling wave fault locator?

II. The Theoretical Basis of Modal Transformations

A. Unsymmetrical transmission line

For the three-wire system shown in fig. 1 using operator calculus the telegraph equations are as follows [11], [12]:

$$-\frac{\partial \underline{\mathbf{U}}}{\partial x} = \underline{\mathbf{Z}}\underline{\mathbf{I}}; \tag{1}$$

$$-\frac{\partial \underline{\mathbf{I}}}{\partial x} = \underline{\mathbf{Y}}\underline{\mathbf{U}}, \tag{2}$$

where $\underline{\mathbf{U}} = \begin{bmatrix} \underline{U}_A(x,p) \\ \underline{U}_B(x,p) \\ \underline{U}_C(x,p) \end{bmatrix}$ – vector of phase complex voltage values;

$\underline{\mathbf{I}} = \begin{bmatrix} \underline{I}_A(x,p) \\ \underline{I}_B(x,p) \\ \underline{I}_C(x,p) \end{bmatrix}$ – vector of phase complex current values;

$\underline{\mathbf{Z}} = \begin{bmatrix} \underline{Z}_A(p) & \underline{Z}_{AB}(p) & \underline{Z}_{CA}(p) \\ \underline{Z}_{AB}(p) & \underline{Z}_B(p) & \underline{Z}_{BC}(p) \\ \underline{Z}_{CA}(p) & \underline{Z}_{BC}(p) & \underline{Z}_C(p) \end{bmatrix}$ – impedance matrix;

$\underline{\mathbf{Y}} = \begin{bmatrix} \underline{Y}_A(p) & \underline{Y}_{AB}(p) & \underline{Y}_{CA}(p) \\ \underline{Y}_{AB}(p) & \underline{Y}_B(p) & \underline{Y}_{BC}(p) \\ \underline{Y}_{CA}(p) & \underline{Y}_{BC}(p) & \underline{Y}_C(p) \end{bmatrix}$ – admittance matrix;

$p=\alpha+j\omega$ – Laplace operator.

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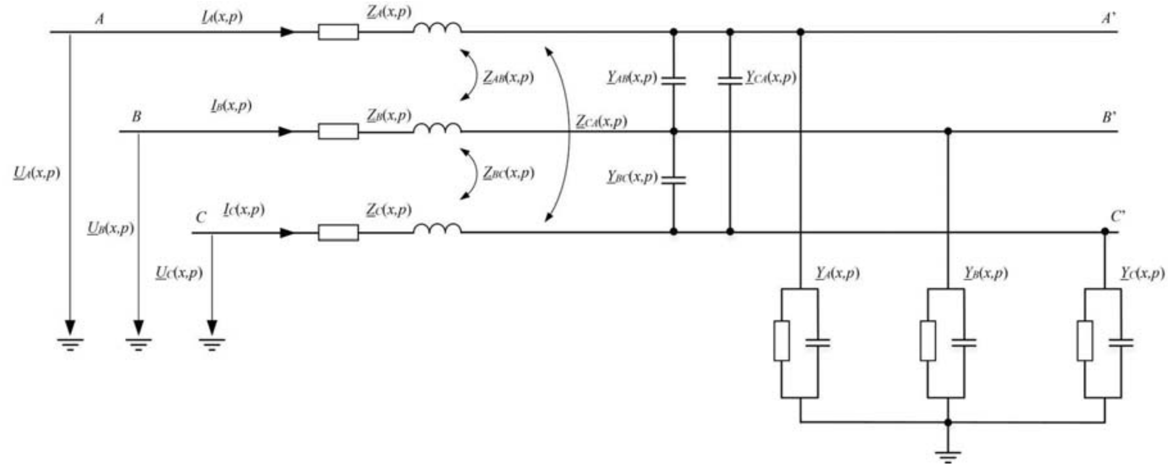

Fig. 1. Infinitesimal three-phase section of the unsymmetrical transmission line

Differentiating both sides of equations (1) and (2), differential equations with one unknown are obtained:

$$\frac{\partial^2 \underline{\mathbf{U}}}{\partial x^2} = \underline{\mathbf{Z}}\underline{\mathbf{Y}}\underline{\mathbf{U}}; \tag{3}$$

$$\frac{\partial^2 \underline{\mathbf{I}}}{\partial x^2} = \underline{\mathbf{Y}}\underline{\mathbf{Z}}\underline{\mathbf{I}}. \tag{4}$$

The solution of differential equations (3) and (4) is a difficult task because it requires taking into account the mutual impedances and admittances of transmission lines [12]. To solve this complexity conveniently to consider wave propagation in modal coordinates in which signals propagate independently of other modes [13].

Modal voltages and currents are determined using the properties of matrix similarity transformations from the equations:

$$\underline{\mathbf{U}} = \mathbf{T}_u \underline{\mathbf{U}}_m, \tag{5}$$

$$\underline{\mathbf{I}} = \mathbf{T}_i \underline{\mathbf{I}}_m, \tag{6}$$

where $\mathbf{T}_u$ and $\mathbf{T}_i$ - matrices of conversion of modal values to phase values.

Differential equations in modal coordinates are obtained by substituting (5) and (6) into (3) and (4) [14]:

$$\frac{\partial^2 \underline{\mathbf{U}}_m}{\partial x^2} = \mathbf{T}_u^{-1} \underline{\mathbf{Z}}\underline{\mathbf{Y}} \mathbf{T}_u \underline{\mathbf{U}}_m; \tag{7}$$

$$\frac{\partial^2 \underline{\mathbf{I}}_m}{\partial x^2} = \mathbf{T}_i^{-1} \underline{\mathbf{Y}}\underline{\mathbf{Z}} \mathbf{T}_i \underline{\mathbf{I}}_m, \tag{8}$$

where $\mathbf{T}_u$ and $\mathbf{T}_i$ chosen so that matrix products both $\mathbf{T}_u^{-1}\underline{\mathbf{Z}}\underline{\mathbf{Y}}\mathbf{T}_u$ and $\mathbf{T}_i^{-1}\underline{\mathbf{Y}}\underline{\mathbf{Z}}\mathbf{T}_i$ are diagonal matrices. This eliminates the occurrence of mutual influence of modes in the system of differential equations (7) and (8).

As is known from the theorem of diagonalizable matrix [12], [15] $\mathbf{T}_u^{-1}\underline{\mathbf{Z}}\underline{\mathbf{Y}}\mathbf{T}_u$ and $\mathbf{T}_i^{-1}\underline{\mathbf{Y}}\underline{\mathbf{Z}}\mathbf{T}_i$ will be diagonal matrices only if matrices $\mathbf{T}_u$ and $\mathbf{T}_i$ consist of eigenvectors of matrices $\underline{\mathbf{Z}}\underline{\mathbf{Y}}$ and $\underline{\mathbf{Y}}\underline{\mathbf{Z}}$ respectively.

Thus, the conversion to modal coordinates comes to the definition of eigenvectors of matrices $\underline{\mathbf{Z}}\underline{\mathbf{Y}}$ and $\underline{\mathbf{Y}}\underline{\mathbf{Z}}$.

B. Symmetrical and transposed transmission line

The problem of definition matrices $\mathbf{T}_u$ and $\mathbf{T}_i$ becomes simpler if assuming that the transmission line is symmetrical and fully transposed as shown in fig. 2. Then

$$\begin{aligned}
&\underline{Z}_A(p) = \underline{Z}_B(p) = \underline{Z}_C(p) = \underline{Z}_L(p);\\
&\underline{Z}_{AB}(p) = \underline{Z}_{BC}(p) = \underline{Z}_{CA}(p) = \underline{Z}_{pp}(p);\\
&\underline{Y}_A(p) = \underline{Y}_B(p) = \underline{Y}_C(p) = \underline{Y}_L(p);\\
&\underline{Y}_{AB}(p) = \underline{Y}_{BC}(p) = \underline{Y}_{CA}(p) = \underline{Y}_{pp}(p).
\end{aligned}$$

Matrices $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ will take the form:

$$\underline{\mathbf{Z}} = \begin{bmatrix} \underline{Z}_L(p) & \underline{Z}_{pp}(p) & \underline{Z}_{pp}(p) \\ \underline{Z}_{pp}(p) & \underline{Z}_L(p) & \underline{Z}_{pp}(p) \\ \underline{Z}_{pp}(p) & \underline{Z}_{pp}(p) & \underline{Z}_L(p) \end{bmatrix}; \tag{9}$$

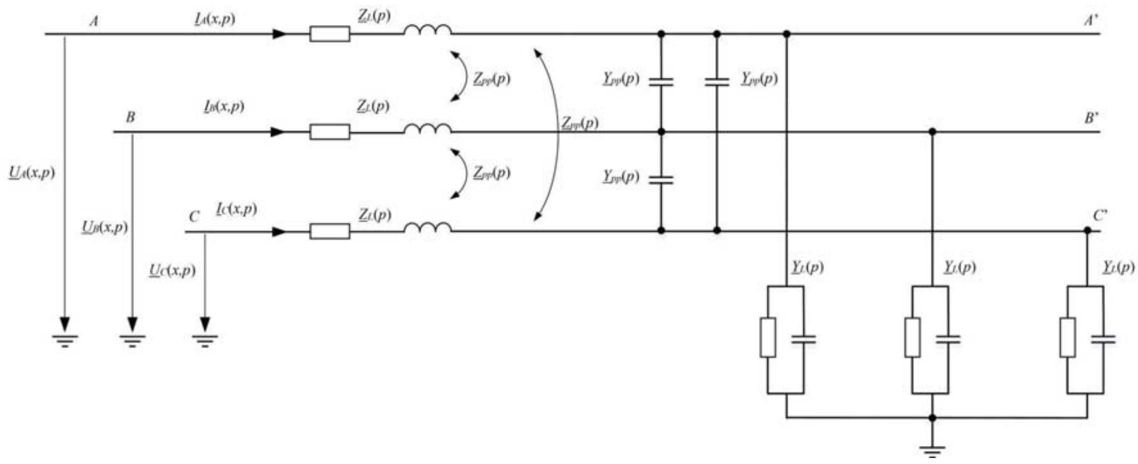

Fig. 2. Infinitesimal three-phase section of the symmetrical transmission line

$$\underline{\mathbf{Y}} = \begin{bmatrix} \underline{Y}_L(p) & \underline{Y}_{pp}(p) & \underline{Y}_{pp}(p) \\ \underline{Y}_{pp}(p) & \underline{Y}_L(p) & \underline{Y}_{pp}(p) \\ \underline{Y}_{pp}(p) & \underline{Y}_{pp}(p) & \underline{Y}_L(p) \end{bmatrix}. \tag{10}$$

Matrices $\underline{\mathbf{Z}}$ and $\underline{\mathbf{Y}}$ are symmetrical, hence, the result of their product will be symmetrical matrix:

$$\underline{\mathbf{Z}}\underline{\mathbf{Y}} = \underline{\mathbf{Y}}\underline{\mathbf{Z}} = \begin{bmatrix} \underline{z}(p) & \underline{m}(p) & \underline{m}(p) \\ \underline{m}(p) & \underline{z}(p) & \underline{m}(p) \\ \underline{m}(p) & \underline{m}(p) & \underline{z}(p) \end{bmatrix}, \tag{11}$$

where $\underline{z}(p) = \underline{Z}_L(p)\underline{Y}_L(p) + 2\underline{Z}_{pp}(p)\underline{Y}_{pp}(p)$;

$\underline{m}(p) = \underline{Z}_L(p)\underline{Y}_{pp}(p) + \underline{Z}_{pp}(p)\underline{Y}_L(p) + \underline{Z}_{pp}(p)\underline{Y}_{pp}(p)$.

To determine the eigenvectors of matrices $\underline{\mathbf{Z}}\underline{\mathbf{Y}}$ and $\underline{\mathbf{Y}}\underline{\mathbf{Z}}$, it is necessary to determine the eigenvalues λ of this matrices:

$$|\,\mathbf{ZY} - \lambda\mathbf{1}\,| = \begin{bmatrix} \underline{z}(p)-\lambda & \underline{m}(p) & \underline{m}(p) \\ \underline{m}(p) & \underline{z}(p)-\lambda & \underline{m}(p) \\ \underline{m}(p) & \underline{m}(p) & \underline{z}(p)-\lambda \end{bmatrix} = 0, \tag{12}$$

where **1** – square identity matrix whose dimension is equal to the dimension of the matrix $\underline{\mathbf{Z}}\underline{\mathbf{Y}}$.

From (12) the eigenvalues of matrices $\underline{\mathbf{Z}}\underline{\mathbf{Y}}$ and $\underline{\mathbf{Y}}\underline{\mathbf{Z}}$ are determined:

$$\lambda_1 = \lambda_2 = \underline{z}(p) - \underline{m}(p);$$
$$\lambda_0 = \underline{z}(p) + 2\underline{m}(p).$$

Find the eigenvector of matrices and $\underline{\mathbf{Y}}\underline{\mathbf{Z}}$, directly following their definition:

$$|\,\mathbf{ZY} - \lambda_i\mathbf{1}\,|\,\mathbf{X}_i = 0, \tag{13}$$

where $\mathbf{X}_i = \begin{bmatrix} X_{i1} \\ X_{i2} \\ X_{i3} \end{bmatrix}$ – eigenvector, i=1, 2, 0.

Considering that the eigenvalues λ_1 and λ_2 are equal, from (13) determine the equalities that the elements of eigenvectors $\mathbf{X}_1$ and $\mathbf{X}_2$ must satisfy:

$$X_{11} + X_{12} + X_{13} = 0; \tag{14}$$

$$X_{21} + X_{22} + X_{23} = 0. \tag{15}$$

The eigenvalue λ_0 corresponds eigenvector with elements:

$$X_{01} = X_{02} = X_{03} = k, \tag{16}$$

where k – arbitrary constant.

Because k is an arbitrary constant, for easement of calculation will accept that k=1. The eigenvector corresponding to eigenvalue λ_0 denoted by the index “0”, given that it is used to calculate the components of zero sequences or ground (zero) modes. For other modes, the indices are “1” and “2” because they are used to calculate the positive and negative sequences or to calculate aerial modes.

III. Comparison of Transformations

Although solutions (14)–(16) have innumerable solutions, all of them can be obtained from

- Clarke transformation

$$\mathbf{T}_C = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 1 & 1 \end{bmatrix};$$

- Karrenbauer transformation

$$\mathbf{T}_K = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix};$$

- Wedepohl transformation

$$\mathbf{T}_W = \frac{1}{3}\begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2} \\ 1 & 1 & 1 \end{bmatrix}.$$

Show that there is a linear correlation between the known transformations.

A. Correlation between Clarke transformation and Wedepohl transformation

The rule of conversion from Wedepohl transformation to Clarke transformation is defined as

$$\mathbf{T}_C = \mathbf{T}_{WC}\mathbf{T}_W,$$

and vice versa

$$\mathbf{T}_W = \mathbf{T}_{CW}\mathbf{T}_C.$$

Conversion matrix from Wedepohl transformation to Clarke transformation

$$\mathbf{T}_{WC} = \mathbf{T}_C\mathbf{T}_W^{-1} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and conversion matrix from Clarke transformation to Wedepohl transformation

$$\mathbf{T}_{CW} = \mathbf{T}_{WC}^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Because this conversion matrices are diagonal, it can be concluded that Clarke and Wedepohl transformations are equal, because it is enough linear coefficient for the mode of one of the transformations to obtain the mode of other transformation.

Thus, Clark and Wedepohl transformations have a linear correlation.

B. Correlation between Clarke transformation and Karrenbauer transformation

The rule of conversion from Karrenbauer transformation to Clarke transformation is defined as

$$\mathbf{T}_C = \mathbf{T}_{KC}\mathbf{T}_K,$$

and vice versa

$$\mathbf{T}_K = \mathbf{T}_{CK}\mathbf{T}_C.$$

Conversion matrix from Karrenbauer transformation to Clarke transformation

$$\mathbf{T}_{KC} = \mathbf{T}_C\mathbf{T}_K^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ \sqrt{3} & -\sqrt{3} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and conversion matrix from Clarke transformation to Karrenbauer transformation

$$\mathbf{T}_{CK} = \mathbf{T}_{KC}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{6} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, in order to obtain from the Clark transformation, the Karrenbauer transformation, and vice versa it is necessary to use a linear combination of aerial modes.

C. Correlation between Wedepohl transformation and Karrenbauer transformation

The rule of conversion from Karrenbauer transformation to Wedepohl transformation is defined as

$$\mathbf{T}_W = \mathbf{T}_{KW}\mathbf{T}_K,$$

and vice versa

$$\mathbf{T}_K = \mathbf{T}_{WK}\mathbf{T}_W.$$

Conversion matrix from Karrenbauer transformation to Wedepohl transformation

$$\mathbf{T}_{KW} = \mathbf{T}_W \mathbf{T}_K^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{3}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and conversion matrix from Wedepohl transformation to Karrenbauer transformation

$$\mathbf{T}_{WK} = \mathbf{T}_{KW}^{-1} = \begin{bmatrix} -1 & \frac{1}{3} & 0 \\ -1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, identically as Clarke transformation, in order to obtain from the Clark transformation, the Karrenbauer transformation, and vice versa it is necessary to use a linear combination of aerial modes.

Thus, there is a linear correlation between all transformations and they do not introduce any special advantages to the operating of a traveling wave fault locator. Therefore, it makes sense to pay attention to the convenience of use one or another transformation in the traveling wave fault locator.

IV. Conclusions

Modal transformation allows splitting aerial and ground modes existing in phase values. Clarke, Karrenbauer and Wedepohl transformations are obtained from modal transformation for symmetrical and fully transposed transmission lines. Due to the fact that all of them have a linear correlation with each other, the characteristics of the traveling waves in all modes formed on the basis of these transformations are identical. Therefore, for fault location any of them can be used – the result of operating a traveling wave fault locator will be the same.

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